EFFECT OF ADDING A NEW EDGE TO A NON-TRIVIAL GRAPH, ON $R_{-1}(G)$ AND HARMONIC INDEX $H(G)$

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Given a graph G, the general Randić index $R_\alpha(G)$, is defined as the sum of the weights $(d(u)d(v))^\alpha$ over all edges uv of G, where $d(u)$ denotes the degree of the vertex u and α is an arbitrary real number. When $\alpha = {}^{-1}/_{2}$, it is defined as the Randić index which was proposed by the Chemist Milan Randić, in 1975. In particular, $R_{-1}(G)$ is one of the most important topological indices investigated by researchers. Another remarkable variant of the Randić index is the Harmonic index, defined as $H(G) = \sum_{uv \in E(G)} \frac{2}{d(u)+d(v)}$, where uv is an edge connecting the vertices u and v. These indices were extensively studied by mathematicians, and several results have been established. Among the studies, it is noticeable that most of the researchers have encountered the effect that edge deletion has on $R_{-1}(G)$ and $H(G)$. In 1998, Bollobás and Erdös first identified the effect of deleting an edge with maximum weight on $R(G)$. In 2013, Mike Cave studied the effect of deleting a non-leaf edge on $R_{-1}(G)$. In a similar manner, many researchers have focused on studying the effect of deleting certain types of vertices and edges on $H(G)$, so that the mathematical background of those indices are well elaborated. In this study, the effect of adding a new edge to a non-trivial graph on $R_{-1}(G)$ and $H(G)$ has been investigated. Also, after adding the new edge, only simple graphs are considered as the resulting graph. Novel upper and lower bounds for$R_{-1}(G+e)$ and $H(G+e)$ have been provided in terms of $R_{-1}(G)$ and $H(G)$ under two main cases, where $G+e$ is the new graph obtained by adding an edge to any non-trivial graph G.

Keywords: Harmonic index, Non-trivial graph, Randić index