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QUASI-COMPACT FOURTH ORDER APPROXIMATIONS FOR FRACTIONAL DERIVATIVES AND APPLICATIONS

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Fractional derivatives (FDs) have recently been used in numerous applications in many branches of science and engineering, including fractal phenomena, anomalous diffusion, viscoelasticity and biological population models. Different types of definitions for FDs have been presented in the literature, of which Riemann-Liouville (R-L) FDs are used in fractional diffusion equations (FDEs). FDs, including R-L FDs, are non-local operators, meaning that an FD at a point involves discrete function values spread throughout the domain. Thus, numerical computation of FDs has become an involved task. Grünwald approximation (GA) derived from Grunwald-Letnikov FDs, equivalent to R-L FDs, is a common approximation for R-L FDs. However, the GA is known to have two main limitations: 1) It is of the first order accuracy and when applied to space-FDEs without shift displays unstable numerical solutions for implicit Euler and Crank-Nicolson(C-N) methods; 2) The shifted form of GA with shift $r = 1$ recovers the stability in these methods but retains the first order accuracy. Therefore, higher order approximations for R-L FDs are of great importance. Recently, extending the notion of GA, a second order Grünwald type approximation with non-zero shift (GA_2), theoretically established as reliable for stability and consistency, was developed for R-L FDs. Then, a third order approximation was constructed using the GA_2. In this study, a quasi-compact operator involving a convex combination of two shifted operators with integer shifts p and q of the GA_2 is defined. Then, for the shift parameters $(p, q) = (1, 0)$ and $(1, -1)$, two fourth order quasi-compact approximations are obtained for R-L FDs. Both the approximations were applied in C-N schemes to solve the one-dimensional space FDE. Numerical results obtained for the preceding schemes confirm the order of accuracy and the convergence of each scheme. However, stability and convergence of the schemes are not theoretically analysed and left for future work.

Keywords: Crank-Nicolson scheme, Fractional derivatives, Fractional diffusion equations Generating functions, Grünwald approximation